## The TMTA Bulletin

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## President's Message

I hope you are doing well as we find our way over half way through the school year. I am a little more than half way through my term as your president. It has been a busy year. We were recently informed that the National Council of Teachers of Mathematics is coming back to Nashville in 2019 for a Regional Conference! The mathematics educators in our region are very active and involved in professional development making Nashville a very popular city for a regional conference. The space and facilities at the new Music City Center and Omni Hotel have made it even better. In addition, NCTM has decided to host the Affiliate Leaders' Conference in Nashville this summer!

As you finish off the spring semester, please let me know if there is anything that I or the Executive Committee can do for you. The Executive Committee's contact information is contained in this bulletin. We are ready and waiting to serve the mathematics education community of Tennessee.
I look forward to seeing you all at the TMTA Annual Conference at the University of Tennessee at Martin in September.

## TMTA Annual Conference Information

## Mathematically Modeling the Standards

Sponsored by:<br>TMTA and co-hosted by MT ${ }^{2}$ NW

## Location:

## University of Tennessee at Martin September 29-30, 2017

Speaker Proposals are currently being accepted. To apply to speak, complete your speaker proposal form at https://tmta. wildapricot.org/Conference. If you have questions about the form or submission process, please contact Kim Mullins at mullinsk@mcnairy.org. If you have general questions about the conference, please contact Stephanie Kolitsch at styler@utm.edu or Desiree McCullough at dmccull1@utm.edu. The speak proposal form is also attached to the end of this newsletter for your convenience.

## TMTA Scholarship Opportunities

## - Dr. Henry Frandsen Scholarship for Teachers

## Criteria:

- applicants must be committed to teaching mathematics in Tennessee at either the secondary or elementary level.
- applicants must have declared an appropriate major at their institution


## Past Winners:

1. 2011: Amber Atkins (MTSU) and Emily McDonald (Tenn. Tech)
2. 2012: Melinda Pierce (UT Knoxville) and Brandy Smith (Austin Peay State University)
3. 2013: Taylor Satterfield
4. 2014: Leanna Ruth Murdoch
5. 2015: Elizabeth Barlow (UT Knoxville)
6. 2016: Courtney Wright (MTSU) and Hillary Grant (UT Knoxville)
7. 2017: Now taking applications at https://tmta.wildapricot.org/page-18062

## TMTA Scholarship Opportunities

## - TMTA Teacher / Scholar Scholarship

## Criteria:

- applicants must be a TMTA member currently teaching in Tennessee and pursuing either a Masters, Ed.S., or doctoral degree to improve their mathematics teaching


## Past Winners:

- 2011: Sarah Hacker (Huntsville Middle School, Scott County)
- 2014: Kathryn Taylor (Northwest High School, Clarksville)
- 2017: Now taking applications at https://tmta.wildapricot.org/page-18062


## TMTA Grant Opportunities

## \$1000 classroom Mini-grant

Criteria to be eligible:

* your school or district must demonstrate financial need;
* you must attend the TMTA Fall Conference to receive your award; and
* you must speak at the next TMTA Fall Conference about your use of the mini-grant.

Application deadline is September 1.

## Past Winners:

- 2013: Tammi Terry
- 2014: Lea Keith
- 2015: Emily McDonald
- 2016: Deana Secrest
- 2017: Now taking applications at https://tmta.wildapricot.org/Grant


## Affiliates

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## Calendar of Events

Middle School Math Contest
High School Math Contest
NCTM Annual Conference
NCTM Leader's Conference
TMTA Mathematics Conference

April 27, 2017
April 11, 2017
San Antonio, TX, April 5-8 ${ }^{\text {th }}$
Nashville, TN, July 24-26
Martin, TN, September 29-30

# Proving the Side-Side-Side congruence criterion by using analytic geometry and Euclid's view of congruence 

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SUMMARY The title expresses the goal that is achieved and the methodology that is used. This note could find classroom use in courses on geometry or Precalculus.

KEYWORDS Euclidean geometry, triangle, congruence criterion, SSS, SAS, neutral geometry, absolute value inequality, circle, square root

## 1. Introduction

Mathematicians have known for more than 100 years that the proof of the Side-Side-Side (SSS) criterion for congruence of triangles in Euclid's Elements [5] is incomplete. In his proof of SSS (in Book I, Proposition 8 of the Elements), Euclid used the Side-Angle-Side (SAS) criterion and the view that figures are congruent precisely when they are superimposable over one another. While Euclid believed that he had proved SAS (in Book I, Proposition 4), virtually all modern approaches to Euclidean geometry (and more generally, to neutral geometry, also known as absolute geometry) contend that Euclid's proof of SAS is incomplete and that one must deal with this gap in the Elements by adding an axiom to Euclid's list of postulates. (Although one can construct some decidedly nonneutral geometries that satisfy all of Euclid's postulates but fail to satisfy SAS, as in [6, pages 92-93], not every modern textbook has shied away from giving Euclid's proof of SSS without a warning: see [7, page 236].) Many textbooks have simply followed Hilbert in adopting SAS as an axiom (cf. [6], [3]), while others have fabricated an axiom that accommodates the above "superimposable" point of view by positing the existence of suitable "motions" that effectively specify a sense of homogeneity in which all the points of a neutral geometry support the same kind of local structure (cf. [1]).

While the logical status of SSS is now secure, universal agreement does not exist as to how this result should be presented to a class. More generally, there has been widespread discussion recently as to whether Euclidean geometry should be developed analytically, synthetically, or via some hybrid. In particular, Harel [4, page 27] has argued that synthetic methods often "provide more insight and understanding" than analytic methods. The purpose of this note is to illustrate the benefits of an analytic or hybrid approach, by giving a proof of SSS that uses analytic geometry in a way that builds on Euclid's view of congruence as superposition. The key step will be to show that two circles of interest actually intersect.
From the proof of the very first result of Euclid (Book I, Proposition 1), this sort of fact is central (although not explicitly proved) in the development of geometry in the Elements.

This note could find classroom use in either a course on geometry or a Precalculus course to reinforce the units on analytic geometry and the solution of inequalities involving absolute value.

## 2. A proof of the SSS

Our task is to prove that if two triangles, $\Delta 1$ and $\Delta 2$, in a Euclidean plane $P$ are such that their corresponding sides are congruent, then $\Delta 1$ and $\Delta 2$ are congruent. We will give a proof which is valid in case $P$ is the usual plane of analytic geometry. That would certainly be enough for a typical class. For an advanced student/reader, one should note that a focus on that particular plane can be done without loss of generality, since the axioms for Euclidean geometry are categorical [2, Proposition 7, page 277], in the sense that any two models (satisfying the axioms) of Euclidean geometry are isomorphic.

Let $r, R$ and $L$ be the side lengths of $\Delta 1$ (and of $\Delta 2$ ). Without loss of generality, $0<r \leq R \leq L$. It is clear that ( ${ }^{*}$ ) $R+L>r$; and $\left({ }^{* *}\right) L+r>R$. Moreover, by what may be termed the "geometric triangle inequality" (Proposition 20 of Book I of the Elements), we also have ( ${ }^{* * *)} r+R>L$. These three inequalities, which jointly characterize the multisets of positive real numbers that can be the multiset of side lengths of a triangle in any neutral geometry, will play key roles toward the end of the proof given below.

Consider the points $A(0,0)$ and $B(L, 0)$. Let $K_{1}$ denote the circle with centre at $A$ and radius $r$; let $K_{2}$ denote the circle with centre at $B$ and radius $R$. We will prove that there is a unique point $C\left(x_{0}, y_{0}\right)$ in the upper half-plane where $K_{1}$ and $K_{2}$ intersect. (It is also true that $K_{1}$ and $K_{2}$ have a unique point of intersection in the lower halfplane, but we shall not need that fact.)

Given the existence and uniqueness of $C$, one can complete the proof as follows. Label the vertices of $\Delta 1$ as $A_{1}$, $B_{1}$ and $C_{1}$ so that the side $A_{1} B_{1}$ has length $L$ and the side $A_{1} C_{1}$ has length $r$. (Then, of course, the side $B_{1} C_{1}$ has length $R$.) Paraphrasing Euclid, "apply" the side $A_{1} B_{1}$ so that its endpoint $A_{1}$ is superimposed on the origin $A$ and its other endpoint $B_{1}$ lies on the positive x -axis. Any approach to neutral geometry grants the existence of a unique point $P$ on the ray $\overrightarrow{A B}$ (i.e., on the positive $x$-axis) such that the segments $A_{1} B_{1}$ and $A P$ are congruent; that is, such that the segment $A P$ has length $L$. Of course, $P=B$. Next, with two of its vertices superimposed/fixed at $A$ and $B$, "apply" $\Delta 1$ so that its other sides $\left(A_{1} C_{1}\right.$ and $\left.B_{1} C_{1}\right)$ "fall into" (i.e., lie in) the upper half-plane. Upon which point $Q$ is the "remaining" vertex $C_{1}$ of $\Delta 1$ superimposed? There is only one possibility, namely, $C$. Indeed, the above "application" (which would nowadays be achieved by a suitable "motion", as in [1]) has superimposed $C_{1}$ upon a point in the upper half-plane whose distance from $A$ is $r$ and whose distance from $B$ is $R$. In other words, $C_{1}$ has been superimposed on a point in the upper half-plane that lies on both $K_{1}$ and $K_{2}$; and, for the moment, we have granted that $C$ is the unique such point. Thus, via the above "applications" or "motions", $\Delta 1$ has been superimposed on the triangle $\Delta 3:=\triangle A B C$. Consequently, from both the classical and the modern perspective, we can conclude that $\Delta 1$ is congruent to $\Delta 3$. In exactly the same way, we see that $\Delta 2$ can be superimposed on $\Delta 3$ and that $\Delta 2$ is congruent to $\Delta 3$. From a modern point of view, it follows that $\Delta 1$ and $\Delta 2$ are congruent, since congruence is an equivalence relation. The same conclusion follows from Euclid's view of congruence. Indeed, the above argument first superimposed $\Delta 1$ on $\Delta$ and then superimposed $\Delta 2$ on $\Delta$. Thus, if we did not "remove" $\Delta 1$ before carrying out the second superpositioning, the effect is to superimpose $\Delta 2$ on $\Delta 1$, which satisfies the classical criterion for $\Delta 1$ and $\Delta 2$ to be congruent.

It remains to prove the existence and uniqueness of $C$. Analytically, this means proving that there is a unique solution $\left(x_{0}, y_{0}\right)$ of the system

$$
\left\{\begin{array}{c}
x^{2}+y^{2}=r^{2}  \tag{2}\\
(x-L)^{2}+y^{2}=R^{2}
\end{array}\right.
$$

such that $x_{0} \in \mathbb{R}$ and $y_{0}>0$. (Note that (1) and (2) are Cartesian equations of $K 1$ and $K 2$, respectively.) Subtracting (2) from (1) gives $2 L x-L^{2}=r^{2}-R^{2}$. The solution of this linear equation is the real number $x_{1}=$ $\left(r^{2}-R^{2}+L^{2}\right) /(2 L)$. Using this value for $x$ in (1) (and bearing in mind that $y>0$ in the upper half-plane) leads to $y_{1}=\sqrt{r^{2}-x_{1}^{2}}$. The argument to this stage has proven that there is at most one point in the upper halfplane lying on both K1 and K2. It remains to show that ( $x_{1}, y_{1}$ ) satisfies both (1) and (2) and that $y_{1}$ is a positive (real) number. It is clear that $\left(x_{1}, y_{1}\right)$ satisfies (1). The verification that $\left(x_{1}, y_{1}\right)$ satisfies (2) is longer:

$$
\begin{aligned}
& \left(x_{1}-L\right)^{2}+y_{1}^{2}=\left(\frac{r^{2}-R^{2}+L^{2}}{2 L}-L\right)^{2}+r^{2}-\left(\frac{r^{2}-R^{2}+L^{2}}{2 L}\right)^{2}= \\
& -2 L\left(\frac{r^{2}-R^{2}+L^{2}}{2 L}\right)+L^{2}+r^{2}=-r^{2}+R^{2}-L^{2}+L^{2}+r^{2}=R^{2}
\end{aligned}
$$

The proof is not yet finished because we have not yet shown that $y_{1}>0$. One cannot prove this inequality by simply observing that ( $x_{1}, y_{1}$ ) is on the complex graph of $x^{2}+y^{2}=r^{2}$. Since the complex number $y_{1}$ is a
square root of $r^{2}-x_{1}^{2}$, we must also show that $r^{2}-x_{1}^{2}>0$ (for then and only then can one conclude the existence of a positive square root of $r^{2}-x_{1}^{2}$ ). Equivalently, since $x^{2}$ is a strictly increasing function of the positive real variable $x$, we must show that $\left|x_{1}\right|<r$, that is, that

$$
\frac{\left|r^{2}-R^{2}+L^{2}\right|}{2 L}<r
$$

Equivalently, we must show that $\left|r^{2}-R^{2}+L^{2}\right|<2 r L$. This absolute value inequality is equivalent to the socalled "continued inequality"

$$
-2 r L<r^{2}-R^{2}+L^{2}<2 r L
$$

Proving the two inequalities whose conjunction is the above continued inequality will not involve geometric reasoning per se, but it will make essential use of the inequalities $\left(^{*}\right),\left({ }^{* *}\right)$ and $\left({ }^{* * *}\right)$ that were introduced in the above discussion of the geometric triangle inequality.

For the first of the required inequalities, note that

$$
-2 r L<r^{2}-R^{2}+L^{2} \Leftrightarrow R^{2}<r^{2}+L^{2}+2 r L \Leftrightarrow R^{2}<(r+L)^{2} \Leftrightarrow|R|<|r+L| \Leftrightarrow R<r+L
$$

Thus, this inequality holds by (and, in fact, is equivalent to) ( ${ }^{* *}$ ). Finally, for the second inequality figuring in the above continued inequality, note that

$$
r^{2}-R^{2}+L^{2}<2 r L \Leftrightarrow r^{2}-2 r L+L^{2}<R^{2} \Leftrightarrow(r-L)^{2}<R^{2} \Leftrightarrow|r-L|<R .
$$

So, the inequality in question is equivalent to the continued inequality $-R<r-L<R$; that is, equivalent to the conjunction " $-R<r-L$ and $r-L<R$ ". This conjunction is, of course, equivalent to the conjunction of $\left(^{* * *}\right)$ and $\left({ }^{*}\right)$. This completes the proof.

## References

[1] A. Barager, A survey of classical and modern geometries: with computer activities, Prentice-Hall, Englewood Cliffs, 2001.
[2] K. Borsuk and W. Szmielew, Foundations of geometry, North Holland, Amsterdam, 1960.
[3] M. J. Greenberg, Euclidean and non-Euclidean geometry development and history, 2nd ed., Freeman, San Francisco, 1980.
[4] G. Harel, Common sense standards for geometry: an alternative approach, Notices Amer. Math. Soc., 61 (2014), 24-35.
[5] T. L. Heath, The thirteen books of Euclid's Elements, second ed., three volumes, Dover, New York, 1956.
[6] E. E. Moise, Elementary geometry from an advanced standpoint, McGraw-Hill, Reading, Mass., 1963.
[7] S. S. Stahl, A gateway to modern geometry the Poincaré half-plane, second ed., Jones and Bartlett, Boston, Mass., 2008.

If you would like to share information, lesson plan ideas, or tips for instruction, please email Lisa Elliott at Lisa.Elliott@cmcss.net.

Are you pursuing an advanced degree to improve your mathematics teaching? There are scholarship funds available to support your learning!

The Tennessee Mathematics Teachers Association (TMTA) provides a $\$ 1000$ scholarship and free membership in TMTA for Year 1 to individuals who meet the following criteria:
$\checkmark$ Current teacher in Tennessee
$\checkmark$ Pursuing a Masters, Educational Specialist (Ed.S), or Doctoral degree to improve your mathematics teaching
$\checkmark$ TMTA member

All you need to do is click on this link: Scholarship Application Form (PDF File) and follow the directions on the application.

The deadline for the application is June 1. Don't delay!
Access the application at the above link and you are on your way!


Should I Pursue a Master's Degree?
Contributed by: Rebecca Darrough
Yes! A Master's degree in the field of Mathematics Education can provide an individual with both personal and professional benefits. Earning a Master's degree offers an individual the opportunity to increase one's personal growth, as well as to provide the individual with a sense of accomplishment. The process of working on a graduate degree increases problem-solving skills, critical thinking, and technical skills. Development of these skills along with the additional mathematics content received during the program gives students a greater understanding of mathematics content and pedagogy, which then can lead to better teaching practices. In addition to these internal qualities, obtaining a graduate degree provides professional benefits including: better opportunities for leadership, greater career advancement, improved employment opportunities, and increased financial benefits.
According to Gallagher (2014), graduate programs that are grounded in the professional workplace and that are offered in executive, online, or hybrid formats are considered innovative educational programs. Austin Peay State University (APSU) provides such a program. Middle school and high school educators can earn their Master's degree in Curriculum and Instruction with a Mathematics Specialization in two years while continuing to work full-time. The program includes courses that are hybrid (online and face-to-face) and online only. On the following page is a listing of the offered courses for the middle school and high school level with details on dates the courses will be offered. If you think that you are ready to earn your Master's degree, contact either Dr. Rebecca Darrough in the Mathematics department at darroughr@apsu.edu or Dr. Loretta Griffy, Associate Provost for Student Success at griffyl@apsu.edu

Martin, D. (2012, June 29). 6 Reasons why graduate school pays off. U.S. News and World Report. Retrieved from http://www.usnews.com
Editorial: Personal benefits of earning a Masters in Education. [Editorial]. Retrieved from http://www.masterseducation.com
Gallagher, S. (2014, April 4). In defense of the Master's Degree. Forbes. Retrieved from
http://www.forbes.com

## Could you be the next Erikkson Graduate Fellow at APSU?

Receive up to $\$ 10,000$ annually to participate in a cohort program to earn a Master of Education in Curriculum and Instruction with a Mathematics specialization at Austin Peay State University. Students will participate in a two-year program. The next cohort begins in June.

## Austin Peay State University (APSU) Master of Education in Curriculum and Instruction - Mathematics Specialization

| MIDDLE SCHOOL EDUCATORS* |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Semester | Course <br> Number | Course Description | Delivery | Face-to-Face Dates |
| $\begin{gathered} \text { Summer } \\ 2017 \end{gathered}$ | MATH 5040 | Number Theory for Elementary and Middle School Teachers | Hybrid | - June 5-9 and June 12-16, 8:30am-3:30pm <br> - July 6 12:30pm-3:30pm <br> - July 7 8:30am-3:30pm |
|  | MATH 5050 | History of Mathematics for Teachers | Hybrid |  |
|  | EDUC 6800 | Seminar on Teaching Effectiveness | Online |  |
| Fall 2017 | MATH 5070 | Methods, Materials, and Strategies in Teaching Mathematics | Online |  |
| $\begin{gathered} \hline \text { Spring } \\ 2018 \end{gathered}$ | MATH 5120 | Contemporary Programs in K-12 Mathematics | Online |  |
| $\begin{gathered} \text { Summer } \\ 2018 \end{gathered}$ | MATH 5080 | Mathematics in a Technological World | Hybrid | - June 4-8 and June 11-15, 8:30am-3:30pm <br> - July 12 12:30pm-3:30pm <br> - July 13 8:30am-3:30pm |
|  | MATH 5060 | Probability and Statistics for Teachers | Hybrid |  |
|  | EDUC 5200 | Evaluation of Teaching and Learning | Online |  |
| Fall 2018 | MATH 5090 | Scientific Writing in Mathematics | Hybrid | TBD |
| $\begin{gathered} \text { Spring } \\ 2019 \end{gathered}$ | MATH 5940 | Research in Mathematics | Hybrid | TBD |

*It may be necessary to make some minor adjustments to the courses in this schedule based on your undergraduate course work. Should this be necessary, your academic advisor will work with you to identify course options within this cohort schedule that best meet your needs.

| HIGH SCHOOL EDUCATORS* |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Semester | Course Number | Course Description | Delivery | Face-to-Face Dates |
| $\begin{gathered} \text { Summer } \\ 2017 \end{gathered}$ | MATH 5520 | Algebra from an Advanced Perspective | Hybrid | - June 5-9 and June 12-16, 8:30am-3:30pm <br> - July 6 12:30pm-3:30pm <br> - July 7 8:30am-3:30pm |
|  | MATH 5640 | Geometry from an Advanced Perspective | Hybrid |  |
|  | EDUC 6800 | Seminar on Teaching Effectiveness | Online |  |
| Fall 2017 | MATH 5070 | Methods, Materials, and Strategies in Teaching Mathematics | Online |  |
| $\begin{gathered} \hline \text { Spring } \\ 2018 \end{gathered}$ | MATH 5120 | Contemporary Programs in K-12 Mathematics | Online |  |
| $\begin{gathered} \text { Summer } \\ 2018 \end{gathered}$ | MATH 5080 | Mathematics in a Technological World | Hybrid | - June 4-8 and June 11-15, 8:30am-3:30pm <br> - July 12 12:30pm-3:30pm <br> - July 13 8:30am-3:30pm |
|  | MATH 5350 | Calculus from an Advanced Perspective | Hybrid |  |
|  | EDUC 5200 | Evaluation of Teaching and Learning | Online |  |
| Fall 2018 | MATH 5090 | Scientific Writing in Mathematics | Hybrid | TBD |
| Spring $2019$ | MATH 5940 | Research in Mathematics | Hybrid | TBD |

## Solutions to the Brainteasers from I/luminations from the Contest Bulletin



Solution: 200 square units; 32 units.
For the first part of the question, the maximum area occurs when the angle between the sides is a right angle. (This is often true for geometry questions involving maximum area: of all quadrilaterals with a given perimeter, the one with maximum area is a square.) When the angle is a right angle, both the base and height of the triangle are 20 units, so the area is 200 square units, which is slightly more than the 192 square units of the original triangle.

For a more advanced trigonometry solution, remember that the area of a triangle can be calculated by taking half the product of two sides and the sine of the angle between those sides. Consequently, the area of any triangle will be greatest when the sine of the included angle is greatest. The sine function reaches its maximum, 1 , when the angle is $90^{\circ}$, so a right triangle will produce the greatest area.

For the second part of the question, note that if you bisect the original triangle, divide it into two right triangles, and rearrange the pieces, you can form a new triangle with exactly the same area. The original triangle had a height of 16 (found through the Pythagorean theorem) and a base of 24 , so its area is $1 / 2(16)(24)=192$ square units. Likewise, the new triangle has a height of 12 and a base of 32 , so its area is also $1 / 2(12)(32)=192$ square units.


Again using a trig solution, $A=\frac{1}{2} a b \sin \theta$, where $a$ and $b$ are the side lengths. For the triangle in this problem, $a=b=20$. Consequently, the area of this triangle is $A=\frac{1}{2} a b \sin \theta=200 \sin \theta$, which has the graph shown below.


The original triangle has an area of 192 units, and the line $y=192$ crosses the graph of $y=200 \sin (\theta)$ at two places when the included angle is $73.74^{\circ}$ or $106.26^{\circ}$. The original triangle has an included angle of $73.74^{\circ}$, and the other triangle with an area of 192 square units has an included angle of $106.26^{\circ}$.

## Speaker Proposal Form: TMTA/MT²NW Annual Conference <br> September 29-30, 2017 Location: University of Tennessee at Martin



